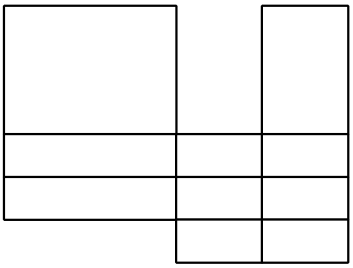
Norfolk Problems for Credit — Summer 2001

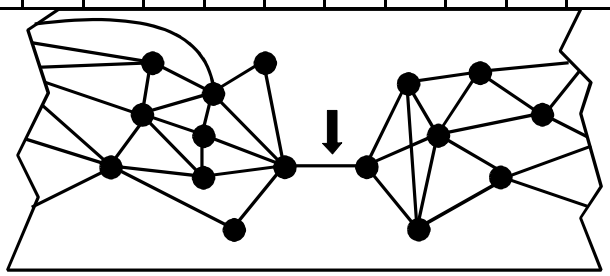
1. To the right is a map whose regions are all rectangular, and two colors suffice to color it.
   a. Can you find a map made of rectangles which requires three colors?
   b. Can you find one which requires four colors?
   c. Five colors?



2. A teacher is organizing a field trip with 12 students, and has 2 large minivans at her disposal with which to transport the students. Unfortunately, some students cannot ride with others, as shown in the "trouble chart" below. Are 2 minivans enough?

	A	B	C	D	E	F	G	H	I	J	K	L
A											X	
B			X									
C		X		X							X	
D			X		X				X			
E				X		X	X					
F					X							
G					X			X	X			
H							X					
I				X			X			X		
J									X		X	X
K	X		X							X		
L										X		

3. A student comes to you one day after discrete math class in order to inform you that you were completely wrong with "that Euler stuff" during the previous class (it was that kind of student). The student then presents a piece of paper as shown to the right, which seems to have been ripped on its left and right sides, and tells this story:



"*I was playing around with Euler circuits, when all of a sudden I found this graph that didn't seem to work with what you taught us yesterday. All the vertices had even degree, and so the graph should have had an Euler circuit. But there was this edge in the middle [she points to it, as shown by the arrow above] which was the only edge connecting the left and right parts of the graph. So, no matter what side you start on, once you cross that edge into the other part, you can never get back, so you can't have a circuit! I was going to bring the graph to school, but the dog ate the left and right sides of the paper.*"

Well, you certainly suspect that this smug student is either lying or mistaken, but can you *prove* it?

4. Sam the sculptor wishes to make a "World Peace" sculpture, and this is his idea: He will sculpt 7 pillars, standing for the 7 continents, and will place them in a circle. Then he will string gold thread between the pillars so that each pillar is connected to exactly 3 other pillars. (Sam explained the reason for the "3," but the explanation is not important here.) Can you see the reason that, although Sam might have good intentions, his sculpture is just physically impossible?

5. In a certain room there are 4 people who like to think of themselves as especially unique! When the introductions start, each is concerned to make sure that they don't shake the same number of hands as anyone else in the room.
    1. Is it possible for these 4 people to each shake a different number of hands than everyone else? If so, how? If not, why not?
    2. How about for 5 people?

6. A girl is putting 75 cents into a vending machine which only accepts nickels and dimes, one coin at a time. In how many different ways can she insert coins to make 75 cents?

7. Bob goes to the ice cream shop to buy Alice and himself sundaes. When he gets there, he realizes that he forgot what sort of sundae Alice wanted! All he remembers is that she wanted 3 different flavors of ice cream (the shop offers 14) and 2 different kinds of topping (the shop offers 5). Now, he knows it is important to Alice that he gets the right sundae, so he decides to get one of each possible type. How many sundaes does poor Bob have to buy?

8. How many 10-digit numbers are there which don't use any digit more than once, where no odd digit ever follows an even digit, and where the "4" and the "5" are next to each other?

9. How can you use the anagram counting method to answer this question, which is problem #14 from yesterday's problem set: *There are 10 employees at a local store, and the manager wishes to assign 1 of them to be the cashier, 2 of them to be cleaners, 3 of them to be "stockers," and 4 of them to be salesmen. In how many ways can this be done?*

10. What is the probability that a hand of 5 cards, randomly dealt from a standard deck, does not contain a "pair," that is, two cards of the same rank?